Packet #1

This unit covers the unit circle, reference angles, coterminal angles, radians, evaluating trigonometric functions along with solving trigonometric equations.

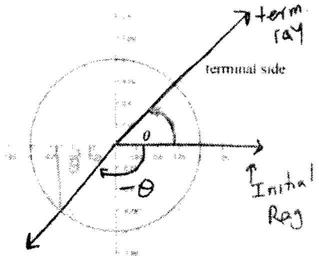
Lesson 1

Angles in Standard Position & Intro to Solving for an Angle without Scientific Calculator

Angles in standard position : vertex at the origin, initial ray is the positive part of x-axis, and there is a terminating ray.

Positive *angles* are generated by counter-clockwise rotation from the initial ray to the terminating ray.

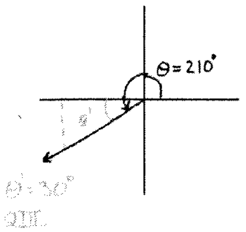
Negative *angles* are generated by clockwise rotation from the initial ray to the terminating ray.



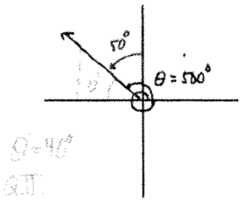
If θ is the original angle in standard position, then the reference *angle*, θ', is the **acute angle** formed by the terminal side of θ and the *nearest* **horizontal axis**. ***Note: A reference angle is always positive.***

Examples. Sketch the angle and identify the quadrant that the terminating ray rests. Also, state the reference angle.

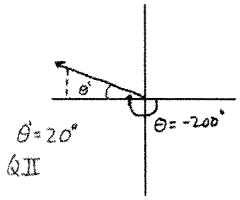
1. $\theta = 210°$



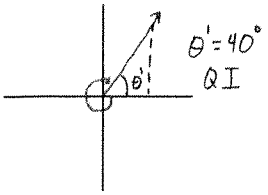
2. $\theta = 500°$



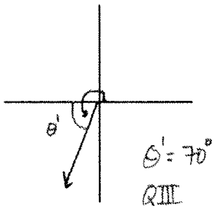
3. $\theta = -200°$



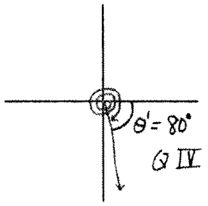
4. $\theta = -320°$



5. $\theta = 250°$



6. $\theta = -800°$



Coterminal Angles and Quadrantals

Coterminal angles are angles in standard position that have the same terminating ray.

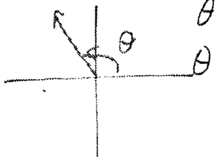
Quadrantals (or *quadrant angles*) are angles in standard position that have a terminating ray that is vertical or horizontal. Basically, quadrantals are straight angles that fall on one of the four axis of the coordinate plane.

**The simplest way to think about coterminal angles are to ±360° to whatever θ is.

Ex 1

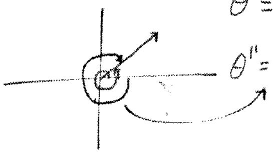
Identify one positive and one negative coterminal angle for the given angle θ.

$\theta = 100$



$\theta' = 460°$
$\theta'' = -260°$

$\theta = 400$



$\theta' = 760°$
$\theta'' = -320°$

Summer Supplement: Trigonometry Unit
Lesson #1: Angles in Standard Position

Name:______________________________

Web Resources

Patrick JMT:

http://patrickjmt.com/coterminal-angles-example-1/
http://patrickjmt.com/coterminal-angles-example-3/

[1-6] Draw each angle in standard position.

1. 80^0
2. 200^0
3. -300^0
4. -70^0
5. 540^0
6. -400^0

[7-12] Identify which quadrant the terminal side of the given angle resides.

7. -45^0
8. 112^0
9. -267^0
10. 480^0
11. -1000^0
12. 632^0

[13-18] Find two angles, one positive and one negative, that are coterminal with the given angle.

13. 30^0
14. -120^0
15. 150^0
16. -300^0
17. 410^0
18. -600^0

[19-30] Draw each angle θ in standard position. Draw a right triangle using the terminal side of the angle and the x-axis as the two sides. Show the reference angle θ' in your diagram and find the measure of θ', the reference angle.

19. 100^0

20. 220^0

21. 310^0

22. -155^0

23. -237^0

24. 412^0

25. -93^0

26. 352^0

27. 194^0

28. 520^0

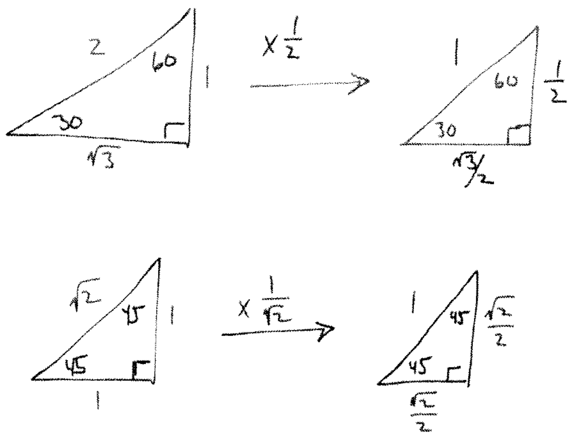
29. -672^0

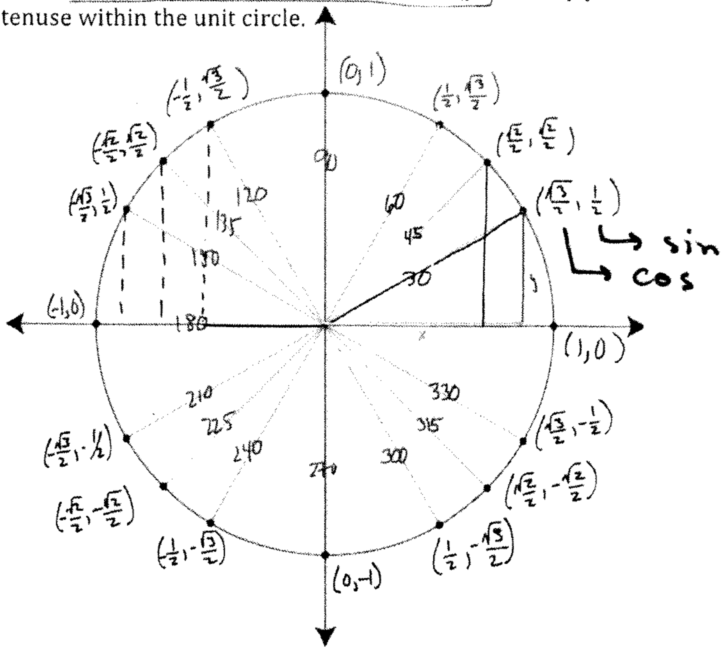
30. 2050^0

Lesson 2

Fill in the missing pieces of the following for the UNIT CIRCLE. Use $\boxed{\sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r}, \text{ and } \tan\theta = \frac{y}{x}}$ to help you fill in the appropriate values. Note: $r = 1$ is always true of the hypotenuse within the unit circle.

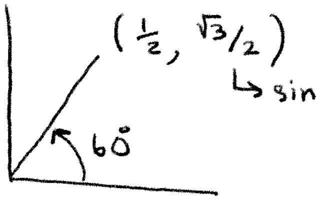
Unit Circle Conversions



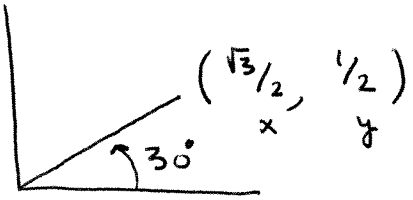


ex 1) Evaluate the trig. function

a) $\sin 60° = \boxed{\sqrt{3}/2}$



b) $\tan 30° = \boxed{\sqrt{3}/3}$



$$* \ \tan\theta = y/x$$

$$= \frac{1/2}{\sqrt{3}/2}$$

* rationalize $$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

Summer Supplement: Trigonometry Unit Name:________________________

Lesson #2: Evaluating Trigonometric Functions: **No Calculator**

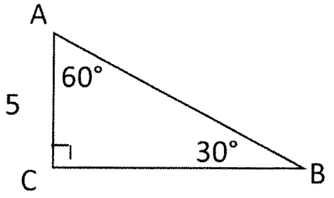
Web Resources

Khan Academy:
https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/unit-circle-definition-of-trig-functions/v/unit-circle-definition-of-trig-functions-1

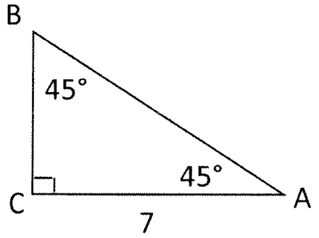
Patrick JMT:
http://patrickjmt.com/special-right-triangles-in-geometry-45-45-90-and-30-60-90/

Math is Fun:
https://www.mathsisfun.com/geometry/unit-circle.html

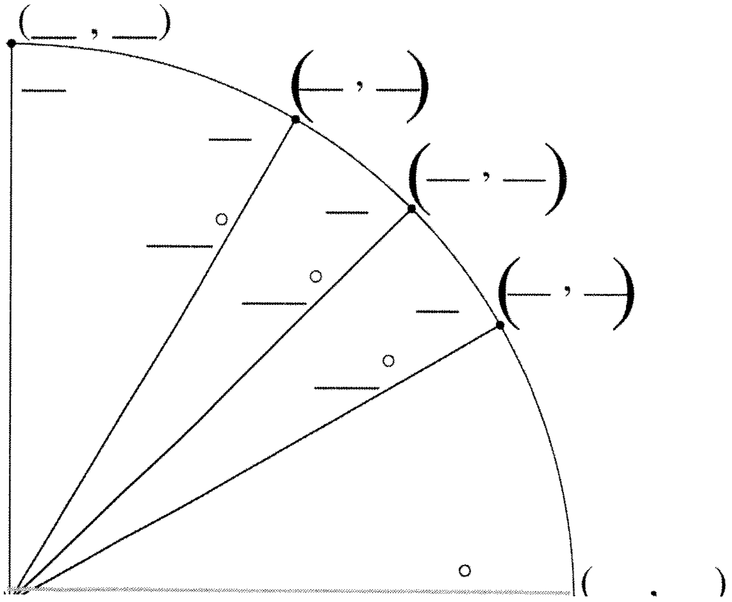
1. Find the sin 30°, cos 60°, tan 60°



2. Find the sin 45°, cos 45°, tan 45°



3. Marley says that if you double the sides of a triangle, you will double the sine and cosine of the angles. Do you agree or disagree with Marley? Include an example to support your answer.



θ	0°	30°	45°	60°	90°
Cos θ					
Sin θ					
Tan θ					

Use the unit circle to the left to complete the table above.

These values will need to be memorized. The back side of this worksheet is memorization practice. Study this table and the unit circle. Some people find it easier to memorize by picturing the unit circle. Others memorize patterns in the table. After studying, try the back without peeking. Check and correct your answers using the information on this side.

Give the value of each trig function at the give degree measure.

1. Sin 30^0

2. Cos 45^0

3. Sin 45^0

4. Sin 60^0

5. Cos 60^0

6. Cos 90^0

7. Sin 90^0

8. Cos 60^0

9. Cos 0^0

10. Sin 0^0

11. Tan 0^0

12. Tan 45^0

13. Sin 30^0

14. Cos 30^0

15. Tan 30^0

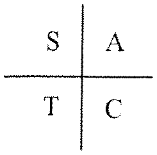
16. Cos 45^0

17. Sin 60^0

18. Tan 60^0

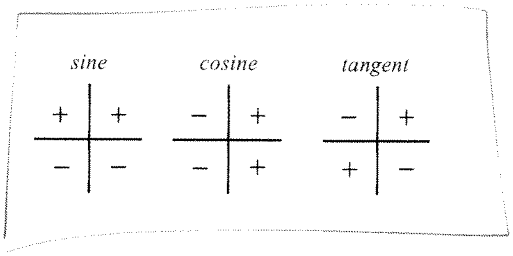
Lesson 5

It is important to note what the positivity of each trig function is given the quadrant of the reference angle. You may have noticed a pattern in the problems above. Here are two charts that help summarize this idea.



Which means that *All* trig functions are positive in Q1. *Sine* is positive in Q2. *Tangent* is positive in Q3. And finally, *Cosine* is positive in Q4.

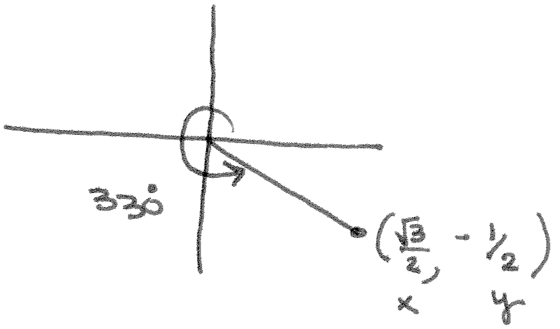
OR



More examples:

Sketch each angle in standard position, identify the quadrant for the terminating ray, identify the reference angle, find $\sin\theta$, $\cos\theta$, *and* $\tan\theta$.

$\theta = 330°$



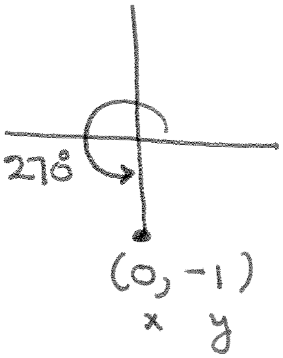
$\sin 330° = -\frac{1}{2}$

$\cos 330° = \frac{\sqrt{3}}{2}$

$\tan 330° = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= -\frac{\sqrt{3}}{3}$

$\theta = 270°$



$\sin 270° = -1$

$\cos 270° = 0$

$\tan 270° = -\frac{1}{0}$

= undefined

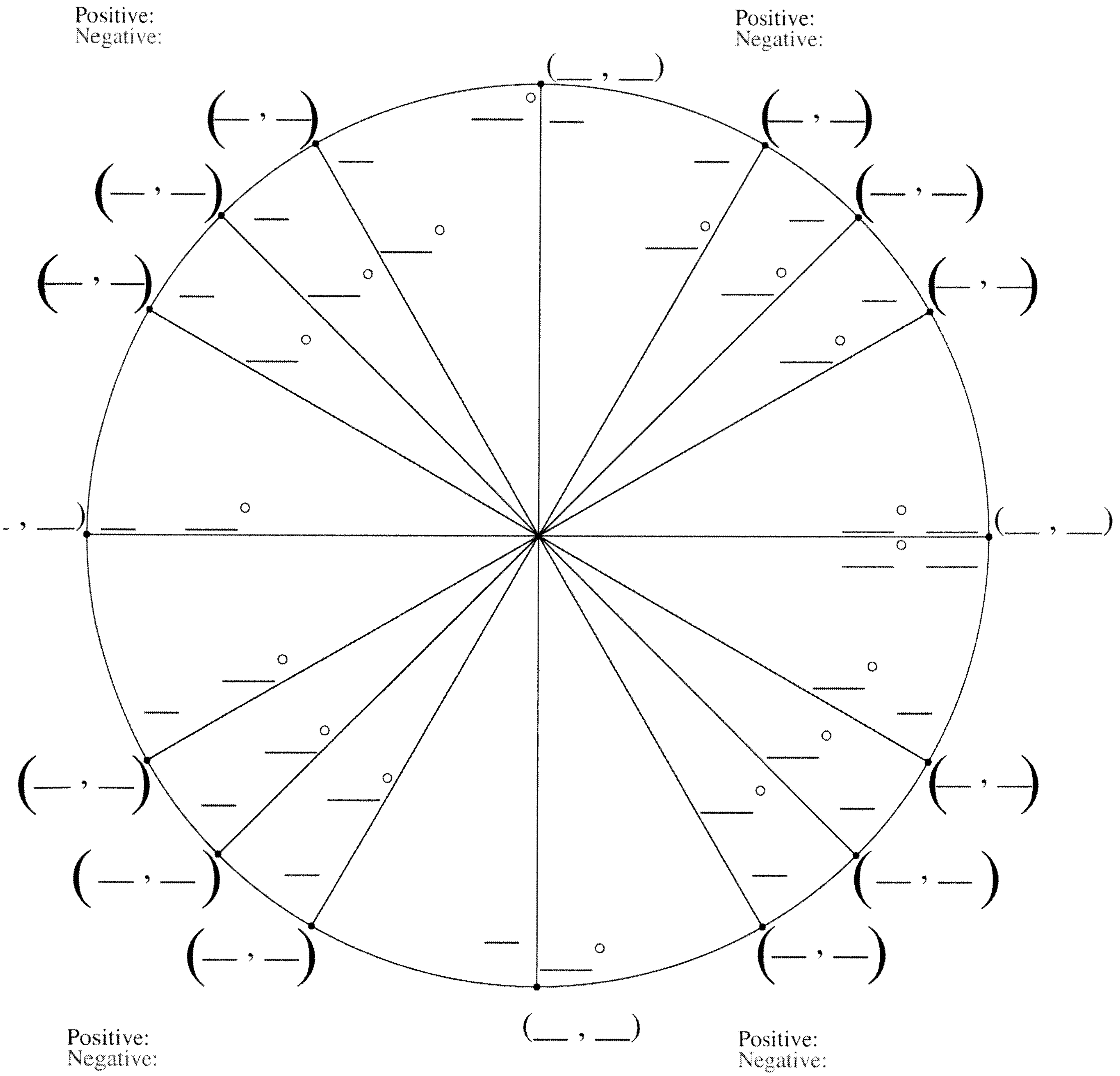
Summer Supplement: Trigonometry Unit
Lesson #3: The Unit Circle: **No Calculator**

Name:______________________________

Web Resources

Math is Fun: https://www.mathsisfun.com/geometry/unit-circle.html

Label all angles in degrees. Label each point on the unit circle.



Sketch the angle in the correct quadrant and then give the value for the specified trig function using the unit circle.

Complete this side from memory. After completing this section, check your answer from the unit circle.

1. cos 30°
2. sin 45°
3. cos 90°
4. cos 60°
5. sin 60°
6. sin 30°
7. cos 120°
8. sin 135°
9. sin 120°
10. sin 150°
11. cos 135°
12. sin 180°
13. cos 180°
14. sin 225°
15. cos 240°
16. sin 240°
17. cos 210°
18. sin 270°
19. cos 300°
20. cos 315°
21. sin 315°
22. sin 300°
23. sin 360°
24. cos 360°

Sketch the angle in the correct quadrant and then give the value for the specified trig function using the unit circle.

25. tan 30°
26. tan 45°
27. tan 60°
28. tan 90°
29. tan 135°
30. tan 150°
31. tan 180°
32. tan 225°
33. tan 270 °
34. tan 315°
35. tan 330°
36. tan 360°

Summer Supplement: Trigonometry Unit Name:______________________________

Lesson #4: All Six Trig Functions: **No Calculator**

Web Resources:

Patrick JMT:

http://patrickjmt.com/finding-trigonometric-function-values-given-one-trig-value-in-a-right-triangle-ex-1/

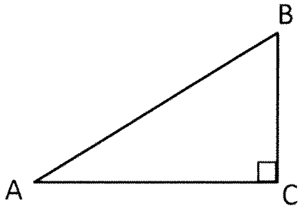
[1-3] Use ΔABC. Find the value of all six trig functions for the given angle. Rationalize the denominators.

1.

$sinA = \frac{3}{5}$ $cscA =$

$cosA =$ $secA =$

$tanA =$ $cotA =$



2.

$sinB =$ $cscB =$

$cosB = \frac{\sqrt{39}}{8}$ $secB =$

$tanB =$ $cotB =$

3.

$sinA =$ $cscA =$

$cosA =$ $secA =$

$tanA = \frac{4}{5}$ $cotA =$

Evaluate the trig function. Be sure to show your work.

4. sec 30^0=

5. csc 30^0=

6. cot 30^0=

7. sec 60^0=

8. csc 60^0=

9. cot 60^0=

10. sec 45^0=

11. cot 45^0=

12. sec 0^0=

13. csc 0^0=

14. cot 0^0=

15. sec 90^0=

16. csc 90^0=

17. csc 135^0=

18. csc 300^0=

19. csc 180^0=

20. sec (-150^0)=

21. sec 360^0=

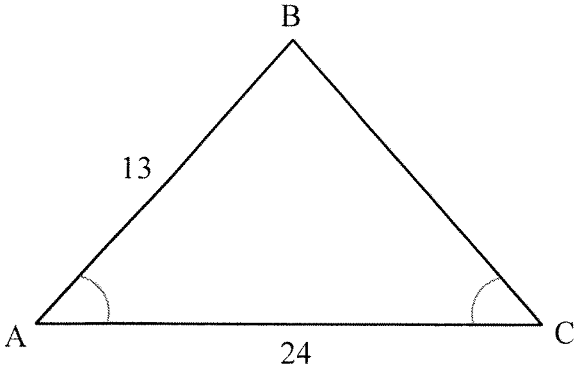
22. cot 270^0=

23. cot (-150^0)=

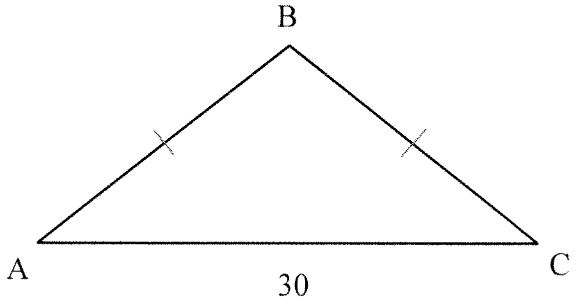
24. cot 120^0=

Use your geometry knowledge to create a right triangle and find the lengths needed to solve the problem.

25. Find the ratios for $sinA$, $cosA$, $tanA$.



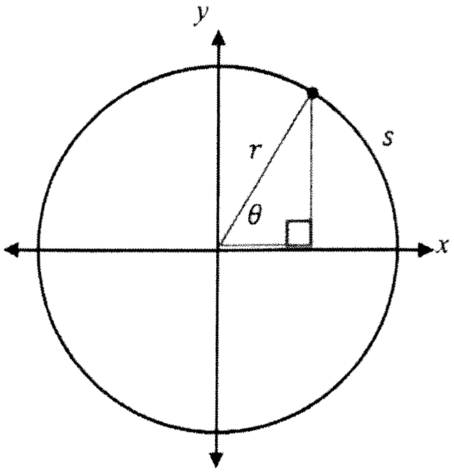
26. If $anA = \frac{1}{3}$, find the perimeter and area of ΔABC.



Lesson 5

- Radian Measure

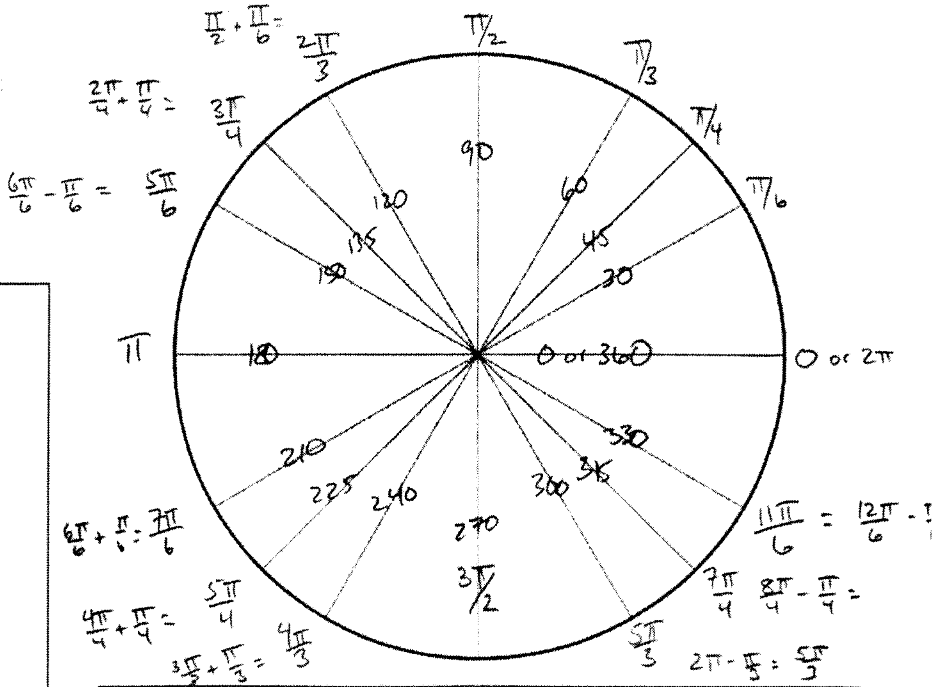
For the last few days we have been working with angles using degree measure; however, degrees do not take into account the size of the radius for the circle and therefore do not give us the length of the arc associated with the angle. Radian measure for angles does since it is defined as $s = r\theta \ \ or \ \ \theta = \frac{s}{r}$, where s is the length of the arc intercepted by the angle θ, and r is the radius for the circle associated with this angle in standard position.



Since radian measure is new for us, we need to relate it to degree measure so we can understand it. We are dealing with a circle, and we know that the circumference of a circle is $c = \pi d \ \ or \ \ 2\pi r$. We will use the $c = 2\pi r$, since radian measure has a radius involved in the formula as well.

Using the formula from the definition of radian measure, $s = r\theta$, if we choose an arc length, s, that is the circumference of the circle, then we can substitute the formula for the circumference of the circle for the arc length s. In other words, $s = r\theta \ \rightarrow \ 2\pi r = r\theta$.

If we divide both sides of the equation by r, the result is $2\pi = \theta$. Remember we chose an arc length, s, that is the circumference of the circle. This means that the angle, θ, that matches up with that particular arc length is 360°. So we now have a relationship between degrees and radians, $360° = 2\pi \ radians$.



Let's set up a table of common radian values with which we need to be very familiar and friendly.

$360° = 2\pi \ radians$

$180° = \underline{\ \pi\ } \ radians$

$90° = \underline{\ \pi/2\ } \ radians$

$60° = \underline{\ \pi/3\ } \ radians$

$45° = \underline{\ \pi/4\ } \ radians$

$30° = \underline{\ \pi/6\ } \ radians$

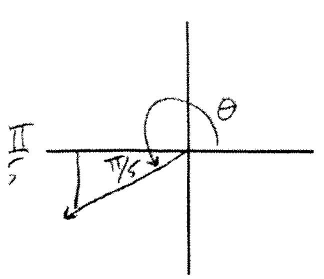
We still need to be able to sketch angles in standard position, know which quadrant the terminating ray is in, and be able to identify the reference angle.

We also want to continue using our 3 trig functions in terms of *SohCahToa* **and** $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, & $\tan\theta = \frac{y}{x}$.

Ex. 1

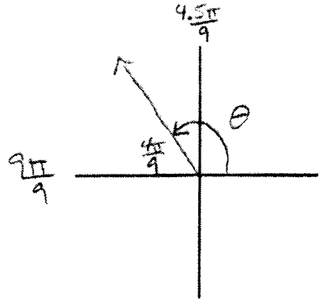
Sketch each angle in standard position, identify the quadrant in which the terminating ray lies, and identify the reference angle.

1. $\theta = \frac{6\pi}{5}$



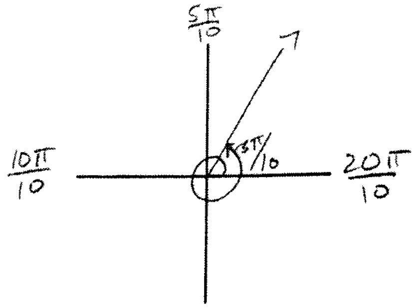
Q3

2. $\theta = \frac{5\pi}{9}$



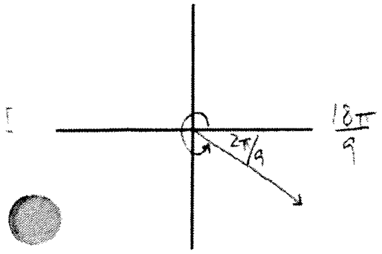
Q2

3. $\theta = \frac{23\pi}{10}$



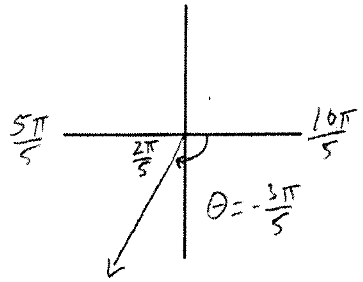
Q1

4. $\theta = \frac{16\pi}{9}$



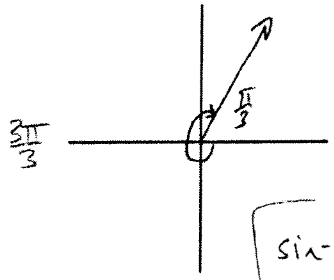
Q4

5. $\theta = -\frac{3\pi}{5}$



Q3

6. $\theta = -\frac{5\pi}{3}$



Q1

$\sin -\frac{5\pi}{3} = \frac{\sqrt{3}}{2}$ $\tan -\frac{5\pi}{3} \cdot \frac{1}{\sqrt{3}}$

$\cos -\frac{5\pi}{3} = \frac{1}{2}$

Conversions of radians and degrees as follows:

Radians to Degrees: multiply by $\frac{180°}{\pi}$

Degrees to Radians: multiply by $\frac{\pi}{180°}$

Convert the following angles from radians to degrees or from degrees to radians.

$\frac{7\cancel{\pi}}{\cancel{10}} \cdot \frac{\cancel{180°}}{\cancel{\pi}} = \boxed{126°}$

$\frac{96°}{1} \cdot \frac{\pi}{180°} = \frac{96\pi}{180} = \boxed{\frac{8\pi}{15} \text{ radians}}$

$\frac{9\pi}{13} \cdot \frac{180°}{\pi} = \frac{1620}{13} = \boxed{124.6°}$

$\frac{250°}{1} \cdot \frac{\pi}{180°} = \frac{250\pi}{180} = \boxed{\frac{25\pi}{18}}$

Summer Supplement: Trigonometry Unit

Name:__________________________

Lesson #5: Radians: **No Calculator**

Web Resources

Khan Academy:

https://www.khanacademy.org/math/geometry/cc-geometry-circles/intro-to-radians/v/introduction-to-radians

https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/intro-to-radians-trig/v/rotation-by-radians-and-quadrants

Patrick JMT:

http://patrickjmt.com/reference-angle-for-an-angle-ex-2-using-radians/

http://patrickjmt.com/degrees-and-radians-and-converting-between-them-example-1/

Math is Fun:

https://www.mathsisfun.com/geometry/radians.html

Draw each angle in standard position using radians. Learn how to do this without converting to degrees.

1. $\frac{\pi}{3}$
2. $\frac{5\pi}{6}$
3. $\frac{2\pi}{5}$
4. $\frac{8\pi}{7}$
5. $\frac{3\pi}{2}$
6. π
7. $\frac{4\pi}{3}$
8. $\frac{11\pi}{6}$
9. $-\frac{7\pi}{4}$
10. $\frac{11\pi}{4}$
11. -5π
12. 4 radians

Find two angles (in radians), one positive and one negative, that are coterminal with the given angle.

13. $\frac{\pi}{6}$
14. $\frac{3\pi}{10}$
15. $-\frac{3\pi}{2}$

Draw each angle θ in standard position. Show the reference angle θ' in your diagram. Find measure of θ' in radians.

16. $\frac{7\pi}{6}$

17. $\frac{2\pi}{3}$

18. $\frac{7\pi}{4}$

19. $-\frac{5\pi}{3}$

20. $\frac{3\pi}{4}$

21. 5 radians

For 22-24, convert to radians. For 25-27, convert to degrees.

22. 100°

23. 235°

24. -330°

25. $\frac{2\pi}{3}$

26. $-\frac{3\pi}{5}$

27. 2 radians

Summer Supplement: Trigonometry Unit Name:______________________
Lesson #6: Evaluating Trig. Functions using Radians: **No Calculator**

Web Resources
Patrick JMT:
http://patrickjmt.com/evaluating-trigonometric-functions-using-the-reference-angle-example-2/

Use the unit circle from Lesson #2 and add in the radian measure for each angle. Evaluate the following.

1. $sin\frac{\pi}{3}$
2. $sin\frac{\pi}{6}$
3. $sin\frac{\pi}{4}$
4. $sin\frac{\pi}{2}$
5. $cos\frac{\pi}{3}$
6. $cos\frac{\pi}{6}$
7. $cos\frac{\pi}{4}$
8. $cos\frac{\pi}{2}$
9. $tan\frac{\pi}{3}$
10. $tan\frac{\pi}{6}$
11. $tan\frac{\pi}{4}$
12. $tan\frac{\pi}{2}$
13. $sin\frac{2\pi}{3}$
14. $cos\frac{7\pi}{6}$
15. $tan\frac{3\pi}{4}$
16. $sin\frac{\pi}{2}$
17. $\cos\pi$
18. $tan\frac{3\pi}{2}$
19. $sin\frac{11\pi}{6}$
20. $sec\left(-\frac{\pi}{4}\right)$
21. $tan\left(-\frac{5\pi}{6}\right)$
22. $csc\frac{5\pi}{4}$
23. $cos\frac{2\pi}{3}$
24. $\cot(2\pi)$
25. $sin\frac{11\pi}{4}$
26. $cos\frac{21\pi}{6}$
27. $tan\frac{15\pi}{6}$
28. $\sin(5\pi)$
29. $cos\frac{9\pi}{2}$
30. $\tan(-\frac{11\pi}{3})$

Give the point on the unit circle that corresponds to the given angle.

31. $\theta = \frac{3\pi}{4}$
32. $\theta = -\frac{2\pi}{3}$
33. $\theta = \frac{3\pi}{2}$

Summer Supplement: Trigonometry Unit Name:__________________________

Lesson #7: Evaluating Trig. Functions at the Quadrantals: **No Calculator**

Web Resources:

https://www.youtube.com/watch?v=DO8DoxwLy8k

Evaluate the trig function at the given quadrantal angle.

1. $\cos\pi$

2. $\cos\frac{\pi}{2}$

3. $\sin\frac{3\pi}{2}$

4. $\sin 90°$

5. $\sin(-180°)$

6. $\sin(-\frac{\pi}{2})$

7. $\sec -270°$

8. $\sec\pi$

9. $\tan\frac{\pi}{2}$

10. $\cot 90°$

11. $\sec 0$

12. $\csc\frac{3\pi}{2}$

13. $\cot 180°$

14. $\csc\frac{\pi}{2}$

15. $\tan(-2\pi)$

Review: Draw the given angle in the correct quadrant, then find the value of sin, cos and tan for the given angle.

16. $\theta = 120°$

17. $\theta = -330°$

18. $\theta = -225°$

19. $\theta = 300°$

20. $\theta = 750°$

21. $\theta = -495°$

22. $\theta = 510°$

23. $\theta = -780°$

24. $\theta = -\frac{5\pi}{6}$

25. $\theta = -\frac{2\pi}{3}$

26. $\theta = \frac{11\pi}{6}$

27. $\theta = -\frac{\pi}{6}$

28. $\theta = -\frac{4\pi}{3}$

29. $\theta = \frac{10\pi}{3}$

30. $\theta = -\frac{20\pi}{3}$

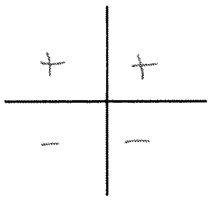
31. $\theta = -7\pi$

Lesson 8

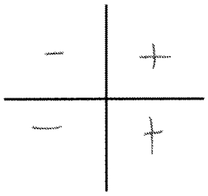
Solving for angles such that $0° \leq \theta < 360°$

When you are solving for all possible solutions for θ between zero degrees and three-hundred and sixty degrees, you will need to take into account whether you are looking for positive or negative quadrants, or both. In objective 10-3, we looked at where sine, cosine, and tangent were positive or negative. Fill in the appropriate positivity for sine, cosine, and tangent.

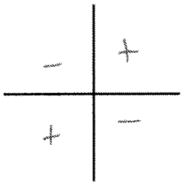
sine $= \frac{y}{r}$



cosine $= \frac{x}{r}$

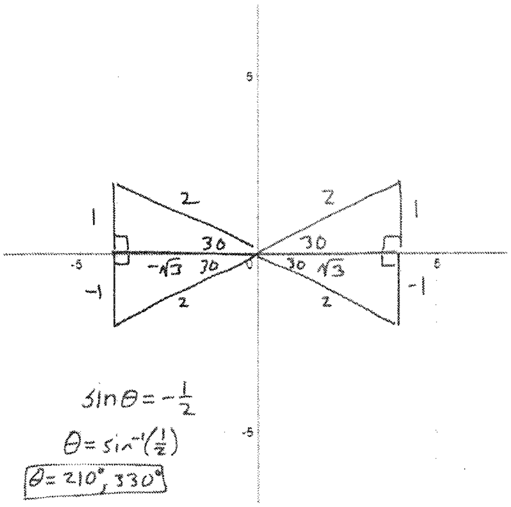


tangent $= \frac{y}{x}$



Now we need to sketch triangles in each quadrant with 30, 60, and 45 degree angles. These values will be paramount for our consideration of what θ can be equal to given certain trig ratios.

30 degree reference angle

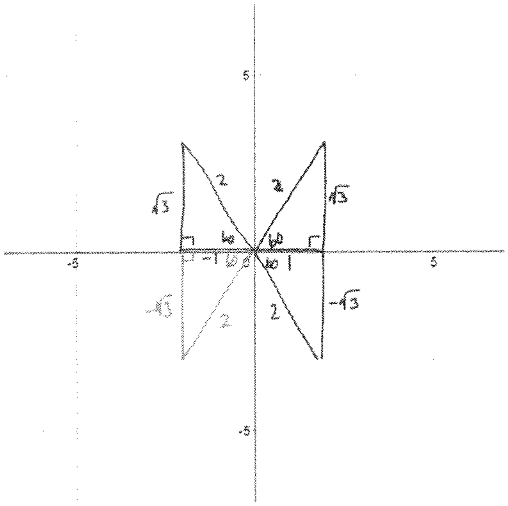


$\sin\theta = -\frac{1}{2}$

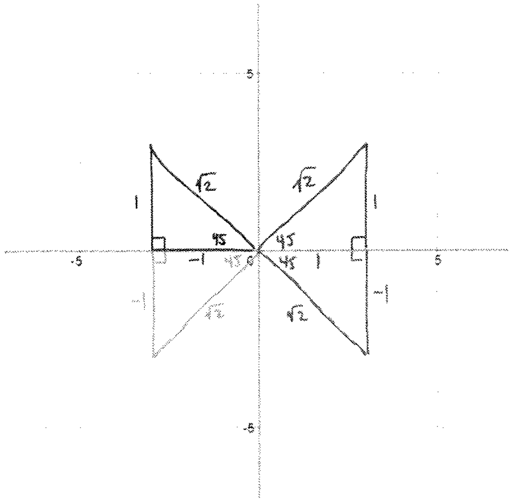
$\theta = \sin^{-1}(\frac{1}{2})$

$\boxed{\theta = 210°, 330°}$

60 degree reference angle



45 degree reference angle



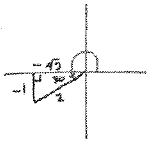
Solve for all values of θ such that $0° \leq \theta < 360°$.

1. $2\sin\theta + 1 = 0$

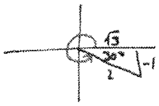
$-1 \quad -1$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$



Q3 ; Q4



30° reference angle

$\theta = \{210°, 330°\}$

2. $6\cos\theta + 3\sqrt{2} = 0$

$-3\sqrt{2} \quad -3\sqrt{2}$

$\frac{6\cos\theta}{6} = \frac{-3\sqrt{2}}{6}$

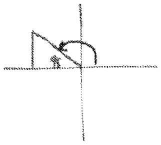
$\cos\theta = -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{4}}{2\sqrt{2}}$

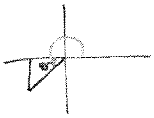
$= \frac{-2}{2\sqrt{2}}$

$\cos\theta = -\frac{1}{\sqrt{2}}$

Q2 ; 3 ⟶ 45° ref

$\boxed{\theta = 135°, 225°}$





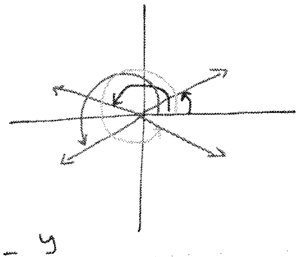
3. $3\tan^2\theta - 1 = 0$

$3\tan^2\theta = 1$

$\sqrt{\tan^2\theta} = \sqrt{\frac{1}{3}}$

$|\tan\theta| = \frac{1}{\sqrt{3}}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ or $\pm\frac{\sqrt{3}}{3} = \frac{y}{x}$



Q1,2,3,4 ⟶ 30° ref

$\boxed{\theta = \{30°, 150°, 210°, 330°\}}$

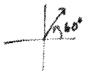
4. $(2\cos\theta - 1)(\tan\theta + 1) = 0$

$2\cos\theta - 1 = 0$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

Q1 ; 4 ⟶ 60° ref.



$\boxed{\theta = 60°, 300°}$

$\tan\theta + 1 = 0$

$\tan\theta = -1$

Q2 ; 4 ⟶ 45° ref.

$\boxed{\theta = 135°, 315°}$

$\boxed{\theta = \{60, 135, 300, 315\}}$

Do you notice any patterns to being able to quickly find θ once you know the reference angle???

Q1 ⟶ θ is the ref. ∠

Q2 ⟶ $\theta = 180 -$ ref. ∠

Q3 ⟶ $\theta = 180 +$ ref ∠

Q4 ⟶ $\theta = 360 -$ ref ∠

How can we get more solutions to the equation, if we extend the boundaries such that $-\infty < \theta < \infty$?

±360 ⟶ coterminal

Summer Supplement: Trigonometry Unit
Lesson #8: Solving Trigonometric Equations: **No Calculator**

Name:________________________

Web Resources:

http://patrickjmt.com/solving-a-basic-trigonometric-equation-example-1/
http://patrickjmt.com/solving-a-basic-trigonometric-equation-example-2/
https://www.youtube.com/watch?v=IE0FxGegdMg

Identify the possible quadrant(s) in which the terminating ray for θ could lie.

1. $\cos\theta > 0$
2. $\sin\theta > 0$
3. $\tan\theta > 0$
4. $\cos\theta < 0$
5. $\sin\theta < 0$
6. $\tan\theta < 0$
7. $cos^2\theta > 0$
8. $sin^2\theta > 0$
9. $tan^2\theta < 0$
10. $tan^2\theta > 0$

For 11-14, solve for all values of θ such that $0° \leq \theta < 360°$. Give all answers in **degrees**.

11. $2\sin\theta - 1 = 0$
12. $2\cos\theta + 1 = 0$

13. $5\tan\theta + 5 = 0$
14. $\sqrt{3}\tan\theta - 1 = 0$

For 15-21, solve for all values of θ such that $0 \leq \theta < 2\pi$. Give all answers in **radians**

15. $2cos^2\theta - 1 = 0$
16. $tan^2\theta - 3 = 0$

17. $4sin^2\theta - 1 = 0$

18. $4cos^2\theta - 3 = 0$

19. $3tan^2\theta + 1 = 0$

20.$(2\sin\theta + \sqrt{3})(\sin^2\theta - 1) = 0$

21. $(\tan\theta + \sqrt{3})(\tan\theta - 1) = 0$

Selected answers: 11. $\{30°, 150°\}$; *15.*$\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$; *21.*$\left\{\frac{2\pi}{3}, \frac{5\pi}{3} \;\&\frac{\pi}{4}, \frac{5\pi}{4}\right\}$