

A rational equation contains at least one rational expression.

Rational Equation

$$\frac{x}{x+1} + \frac{x}{x-1} = 3$$

Not a Rational Equation

$$x + \frac{1}{2} = \frac{2}{3}$$

To solve an equation containing rational expressions, first multiply each side by the least common denominator, LCD, of the rational expressions. Doing this, however, can introduce extraneous solutions. Any time you multiply each side of an equation by an algebraic expression, it is possible to introduce an extraneous solution. An extraneous solution is a solution of the derived equation, but not a solution of the original equation. Checking for excluded values or substitution in the original equation confirms the solutions.

[1-2] Solve the following rational equations and check for extraneous solutions.

$$1. \quad \frac{x}{x-2} - \frac{2}{x-4} = \frac{4x-12}{x^2-6x+8}$$

$$2. \quad \frac{x}{x-1} - \frac{2}{x} = \frac{1}{x-1}$$

Steps

1. Find the common denominator (lowest is best)

*Factor first!

*Find excluded values from LCD

2. Multiply each side by the common denominator to clear the fractions.

3. Simplify and solve.

- Factor
- Isolation
- Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4. Check your solutions. Are any extraneous? Are any excluded values?

[3-8] Solve the following equations and then state whether each equation has **no real solution**, **one real solution**, **two real solutions**, or **infinitely many solutions**.

$$3. \frac{x}{x+1} + \frac{3}{x+4} = \frac{x+3}{x+4}$$

$$4. \frac{5x}{4} - \frac{3}{x} = \frac{1}{4}$$

$$5. \frac{x^2-3}{x+2} = \frac{x-3}{2}$$

$$6. \frac{2}{x} + \frac{x+2}{x+1} = \frac{-2}{x^2+x}$$

$$7. \frac{x}{x^2-6} = \frac{1}{x}$$

$$8. \frac{2}{3x+1} = \frac{6}{9x+3}$$